An Interactive Tool for the Synthesis of Consonant Timbres Based on Dissonance Models

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Abstract. This article describes the implementation of an interactive software patch for Pure Data (Pd) that aims to generate consonant timbres by selecting and manipulating spectra partials using dissonance models to improve the consonance between intervals. First, we describe the technique used to create spectra based on Sethares' and Vassilakis' models of sensory dissonance. Then, we propose a method for reducing dissonance by designing an appropriate spectra for a given target equal tempered scale. A patch to generate the timbre sounds is presented, which can be played using a MIDI keyboard. At last, we show an example of a timbre spectrum generated by our patch and validate the method efficacy by inspecting its corresponding dissonance curve.

Keywords: Consonant timbres; Dissonance models; Psychoacoustics; Equal tempered scales;

1 Introduction

Dissonance is a multidimensional attribute of the sound that can be approached through many ways, considering cultural aspects, the musical practice, and the physical properties of the sound. Tenney [4] divides those approaches in five different categories: melodic, polyphonic, functional, contrapunctual and psychoacoustics. In the present research, we do not consider cultural and aesthetic factors, but focus on the concept of psychoacoustic dissonance using a psychophysics approach. Our main motivation is the search for methods to enhance the sense of consonance in both chords and melodic sequences of notes for equal tempered scales, especially for those using a number of divisions rather than the traditional 12-tone chromatic scale.

Helmholtz introduced the concept of roughness in his theory of beats [5], that states that the perception of dissonance is related to the presence of beats between partials. According to this principle, the perception of the highest dissonance between two pure tones occurs when the difference of frequencies between the tones is between 30 Hz and 40 Hz, at any register.

Plomp and Levelt in 1965 [6] reviewed the theory of beats based on the concept of critical bandwidth. Based on a subjective experiment that presented pairs of sinusoidal sounds to volunteers without musical training, they concluded that the roughness sensation happens only in intervals that are situated in the same critical bandwidth, a conception that persists until today. The highest roughness sensation happens at around a quarter of the critical bandwidth, that exhibit different extensions according to the register, being larger at low frequencies and smaller at high frequencies. We can see in fig. 1 a dissonance curve for a 12-step equal tempered scale, resulting from Plomp and Levelt's experiment.

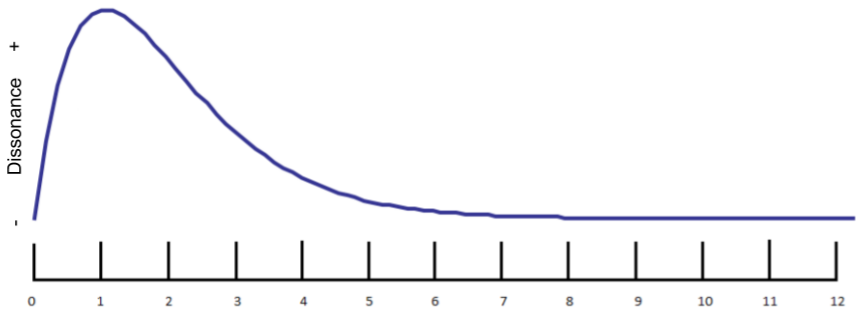

Fig. 1. Dissonance curve obtained by Plomp and Levelt [6]. The x axis represents the pitch span over an octave, indicating the 12 steps of 12-tet scale.

Following our developments pursuing a method for reducing the dissonance between steps of an arbitrary n-step equal tempered musical scale [9], the next sections will lead to the implementation of an interactive patch for Pure Data (Pd) that can synthesize an artificial spectrum, related to a given desired equal tempered scale, that shall exhibit an optimized consonance for a specified number of interval steps.

2 The Dissonance Model

Sethares [2] approximates the dissonance curve of fig.1 with the function on eq. 1.

$$d(x) = e^{-b_1 x} - e^{-b_2 x} \quad \textbf{(1)}$$

where x represents the absolute value of the difference of frequencies between two pure tones and the exponents b_1 and b_2 represent the rates of rise and fall of the curve.

Based on this approximation, Sethares introduces the function of dissonance d (eq. 2), where $f_1 < f_2$, and s is given by eq. 3, where x_m is the point of maximum dissonance and the values of b_1, b_2, s_1 and s_2 are determined from the minimization of quadratic errors by the gradient method [2] of the dissonance curve of figure 1 and are respectively b_1 = 3.5, b_2 = 5.75, s_1 = 0.021 and s_2 = 19.

Sethares' model takes into consideration the *loudness* to estimate the dissonance between partials with different amplitudes. He introduces then the variable l_{12} defined by (4).

$$d(f_1, f_2, l_1, l_2) = l_{12}[e^{-b_1 s(f_2 - f_1)} - e^{-b_2 s(f_2 - f_1)}] \quad \textbf{(2)}$$

$$s = \frac{x_m}{s_1 f_1 + s_2} \quad \textbf{(3)}$$

$$l_{12} = min(l_1, l_2) \quad \textbf{(4)}$$

The dissonance level for a complex sound will be given by the combination of the dissonance levels of each pair of partials. Vassilakis [8] uses the results of subjective tests by Terhardt [7] to substantiate his revision of Sethares' model. This revision affects only the estimation of roughness with pure tones of different amplitudes. Vassilakis' revision was motivated by the fact that Sethares' model overestimates the SPL and underestimates the degree of amplitude fluctuation of the partials. Such revision can be expressed in the following equation:

$$R(f_1, f_2, l_1, l_2) = (l_1 * l_2)^{0.1} * 0.5(\frac{2l_2}{l_1 + l_2})^{3.11}[e^{-b_1 2(f_2 - f_1)} - e^{-b_2 s(f_2 - f_1)}] \quad \textbf{(5)}$$

where $b_1 = 3.5, b_2 = 5.75, s = \frac{x\ \ m}{s_1 f_1 + s_2}, x_m = 0.24, s_1 = 0.0207$ and $s_2 = 18.96$.

3 The Method for Building Consonant Timbre Spectra

Based on the experiences of Sethares [2] and Pierce [1] in the construction of spectrum related to scales, and following the strategies presented in [9], we propose a 3-stage method for the construction of spectra related to an equal tempered scale[1].

The construction of a spectrum related to a scale aims to minimize the sensorial dissonance by reducing roughness sensation between the scale notes through manipulation of the spectrum partials. This procedure should produce an adequate set of partial amplitudes and ratios to the fundamental frequency to be used as a stationary spectrum for the sustain segment of each note of the scale played. Its fundamental frequency and an adequate balance of chosen partials' amplitudes determine the pitch sensation for each note.

The three stages, illustrated in Figure 2, are to set: 1- the ratio of the chosen scale; 2- the chosen scale steps that will be privileged in terms of improved consonance; 3 - the distribution of partials over the critical bands.

[1] By equal tempered scale we mean any scale that divides a given interval - e.g an octave - into *n* equal parts.

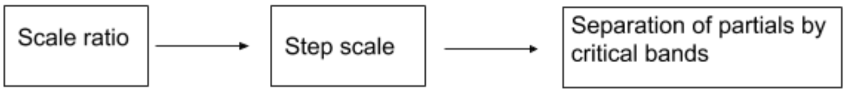

Fig. 2. Steps for the construction of an improved consonant spectrum for a given scale

These tasks aims to minimize the sensorial dissonance in the spectrum of a given *n-tet* equal tempered scale and to enable the choice of consonant dyads in the scale.

4 The *Pure Data* Patch Structure

The next sub-sections present a Pd patch that calculates the partials frequencies to be used in the synthesis of each note for a given n-tet scale, and implements a simple synthesizer to generate the sounds. Its user interface collects some design rules and choices, including a method to define partials amplitudes.

4.1 The front-end of patch interface

Figure 3 shows the front-end interface of the patch. In the field *N-tet of spectra* we determine the equal tempered scale that will generate the spectrum. In the field *Scale Steps* we choose the scale steps candidates to the lowest dissonance levels, from 3 to 5 steps. In the field *N-tet key* we can determine which equal tempered scale will execute the spectrum. In the box *Amplitudes*, we can choose between 4 options of equalization for the partials' amplitudes. At the bottom of the screen, one can see the values of the synthesized spectrum partials and their amplitudes, and the print button will send these values to *Pure Data* terminal.

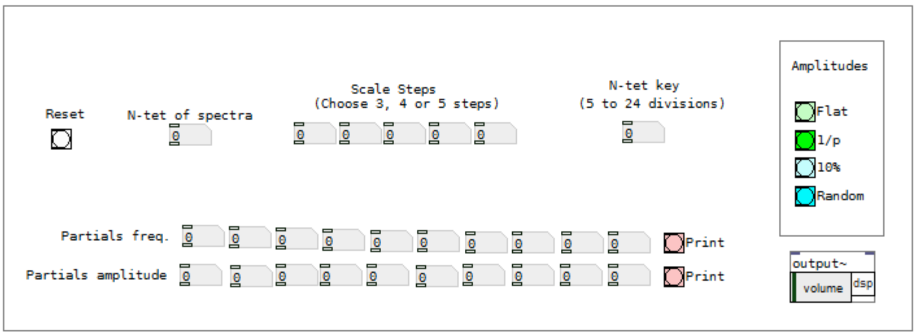

Fig. 3. The front-end of the spectrum generator patch

A synthesizer derived from [9] and presented in topic 4.3 is embedded inside, and an AKAI APC Key 25 MIDI controller, with a two-octave keyboard, has been used to control the patch. The choice of MIDI numbers that control the patch is based on the number of keys available on the keyboard, and its potentiometer is programmed to manipulate the amplitudes of partials.

4.2 The scale generator

The algorithm that generates equal tempered scales uses the corresponding MIDI number of the keyboard to represent a n-tet scale step and sends bangs to start the process. When the bang is activated, the computer multiplies a start frequency of 130.8 Hz by the scale ratio of the chosen equal tempered scale raised to the power *p*, where *p* is the scale step that the MIDI value represents. The computer then converts the note frequency to MIDI so to send the value to the synthesizer. The algorithm is shown in the pseudo-code below.

p ← scale_step;
*note_freq ← (r^p)*fundamental_freq;*
note_midi_value ← note_freq;

4.3 The spectrum generator

The spectrum generator consists in two modules: 1- a sequencer of partials derived from scale steps, and 2- a filter that separates the values sent by the counter according to the critical bandwidths.

To each iteration the sequencer outputs the initial number of the chosen step and the sum ($step + xn$), where x is the number of the iteration and n is the number of divisions of the equal tempered scale (fig 4).

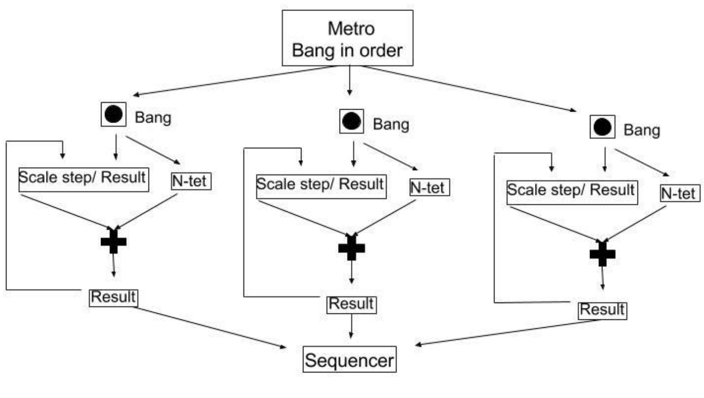

Fig. 4. The sequencer for scale steps in the spectrum generator

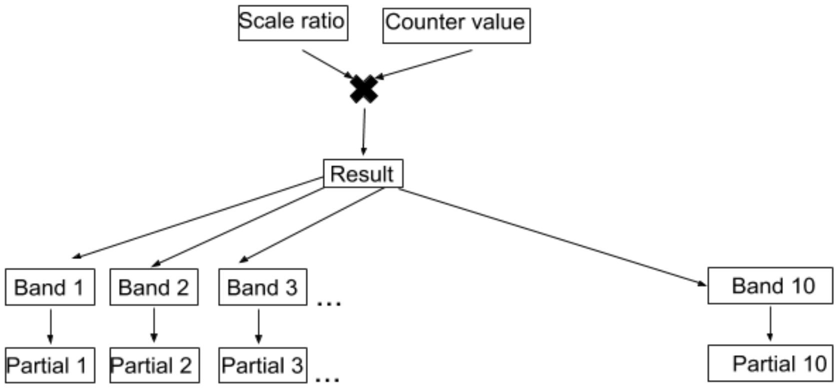

Fig. 5. The band filter of the spectrum generator

Then, the sequencer values are multiplied by the ratio of the chosen equal tempered scale. The corresponding values of the frequencies of the partials are then separated by critical bandwidths filters (fig. 5).

5 The Evaluation of the timbres using dissonance curves

An example spectrum with 8 partials was generated in the patch stated above, with the following specifications:

- Equal tempered scale that will generate the spectrum (*N-tet of spectra*): 8-tet
- Steps of the scale that will be privileged in consonance: 2, 4, 6 and 8.
- Amplitudes in an attenuation of 1/partial

Resulting spectrum:

Partial frequency	F	2.37841F	4F	4.75683F	5.65686F	6.72717F	8F	9.51366F
Partial amplitude	1	0.5	0.332	0.25	0.2	0.165	0.141	0.125

This spectrum with a fundamental frequency of F=261.6 Hz is shown on fig. 6.

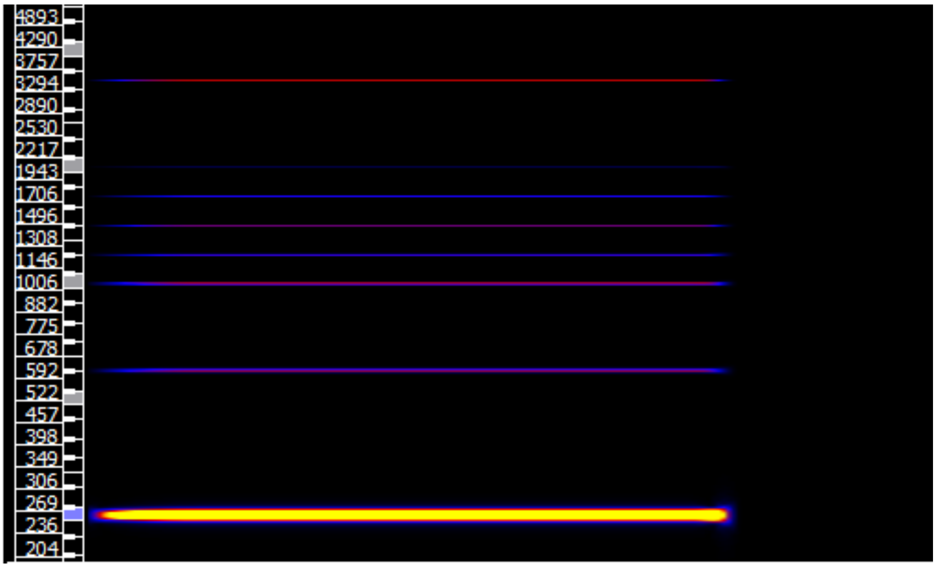

Fig. 6. Spectrogram of the timbre generated by the *Pure Data* patch, done with Sonic Visualiser. The horizontal axis represents time and the vertical, frequency channels. Notice the different partials' amplitudes encoded in color difference.

One can evaluate the efficiency of the method of relating scales and spectra by inspecting the minimum points of dissonance curves. The dissonance curve for this example was generated with *GNU Octave* using Sethares' *MatLab* code and adapting the parameters according to Vassilakis' model[2].

The respective dissonance curve obtained for this spectrum is shown on fig. 7. The curve indicates that the highest levels of consonance are found at the steps 1 and 8, followed by steps 2, 6 and 4. We can also verify that every step of the scale has a correspondent point of consonance. The curve generated by the chosen scale also exhibits a geometric symmetry in terms of local minimums, due to the fact that we have chosen equal distances between the privileged scale steps.

[2] See section 2.

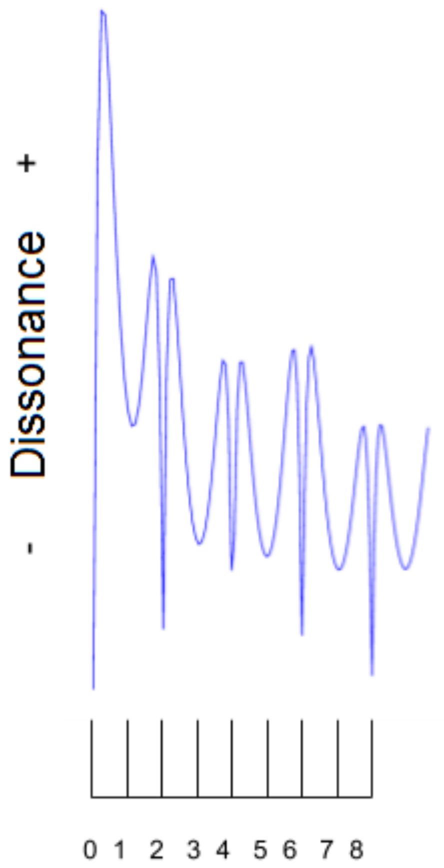

Fig. 7. The dissonance curve obtained for the designed 8-tet spectrum plotted with GNU Octave. The y axis represents the dissonance level and the x axis the frequencies, separated in scale steps.

6 Conclusion and future steps

The *Pure Data* patch was built aiming to create, through an automated process, a spectrum based on the technique of minimization of the dissonance for n-tet scales. Spectra that follow exhibit an intrinsic inharmonic structure. The procedure currently employs a set or rules to calculate partial frequencies and amplitudes, and our implementations have proved useful for testing the theory of sensory dissonance with any scale with 5-24 equal divisions and testing our hypothesis towards modeling it.

The analysis of the obtained spectrum using the dissonance models from Sethares and Vassilakis can confirm the efficacy of the presented method to generate a timbre with improved consonant properties. However, timbres generated using this technique are to be submitted to further subjective auditory tests, so we can test the presented model of consonance with spectra related to equal tempered scales.

Also, considering the non-stationary nature of musical performances, with varying dynamics and expression, it might be interesting to investigate the use of optimization algorithms to calculate adaptive frequency-amplitude ratios for satisfying instantaneous desired levels of dissonance during performance.

In terms of artistic applications, we need to improve the process of synthesis to achieve a time-variant spectra with a more dense texture than simple additive synthesis would permit. Besides that, it is necessary to find other interface (controller) solutions to execute such n-tet scales. A study based on an isomorphic keyboard and digital interfaces is currently under investigation for that.

7 Acknowledgments

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References

1. Pierce, John R. "Attaining consonance in arbitrary scales." The Journal of the Acoustical Society of America vol. 40, n. 1, pp. 249-249 (1966)
2. Sethares, W. A.: Tuning, timbre, spectrum, scale. Springer-Verlag (2005)
3 Blackwood, Easley. "Modes and Chord Progressions in Equal Tunings." Perspectives of New Music vol. 29, n. 2, pp. 166-200 (1991)
4. Tenney, J.: A history of consonance and dissonance. Excelsior (1988)
5. Helmholtz, H. L. F.: On the sensation of tone as a psychological basis for the theory of harmony. Dover Publications (1954).
6. Plomp, R., Levelt, W. J. M.: Tonal consonance and critical bandwidth. Journal of the Acoustical Society of America, vol. 38, n. 4, pp. 548-568 (1965).
7. Terhardt, E.: Pitch, consonance, and harmony. The Journal of the Acoustical Society of America, vol. 55 n. 5, pp.1061-1069 (1974).
8. Vassilakis, P. N.: Perceptual and physical properties of amplitude fluctuation and their musical significance. Doctoral dissertation, University of California, Los Angeles (2001).
9. Silva, M. A.; Faria, R. R. A.: Lowering dissonance by relating spectra on equal tempered scales. In: Proceedings of the 12th International Symposium on Computer Music Multidisciplinary Research, 2016, São Paulo. Marseille: The Laboratory of Mechanics and Acoustics, 2016, pp.323-330 (2016).